

Velocity-dependent energy gaps and dynamics of superfluid neutron stars

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Abstract

We show that suppression of the baryon energy gaps, caused by the relative motion of superfluid and normal liquid components, can substantially influence dynamical properties and evolution of neutron stars. This effect has been previously ignored in the neutron-star literature.

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I. INTRODUCTION

According to numerous microscopic calculations (e.g., [1, 2] and references therein), nucleons and hyperons in the internal layers of neutron stars (NSs) can become superfluid at temperatures $T \lesssim 10^8 \div 10^{10}$ K. Superfluidity has a strong impact on the thermal evolution of NSs, their oscillations, and (most probably) leads to such observational phenomena as glitches [3] and pulsar spin precession [4, 5]. Recent real-time observations [6] of a cooling NS in Cassiopea A supernova remnant give strong arguments that the star has superfluid core [7, 8].

The aim of this short note is to point out the importance of one effect related to superfluidity of baryons in NSs that has usually been ignored in the NS literature. In Sec. II we outline the effect. In Sec. III we demonstrate its efficiency. In Sec. IV we discuss possible consequences for the physics of NSs and in Sec. V we conclude. We use the system of units in which $k_B = \hbar = 1$.

II. A SIMPLE PROBLEM AND THE PROPOSED EFFECT

Let us consider a degenerate Fermi-liquid composed of identical particles. Assume that they interact through a weakly attractive potential so that BCS theory [9] is applicable. Assume also that they pair (become superfluid) in the spin-singlet 1S_0 state at temperatures T below some critical temperature T_c . The role of elementary excitations in such superfluid Fermi-liquid is played by Bogoliubov excitations [10]. In what follows, all equations will be written in a reference frame in which the mean (hydrodynamic) velocity \mathbf{V}_q of Bogoliubov excitations vanishes, $\mathbf{V}_q = 0$ (i.e., normal liquid component is at rest).

In the absence of superfluid current (when the superfluid velocity $\mathbf{V}_s = 0$) the energy $E_{\mathbf{p}}$ of a Bogoliubov excitation with momentum \mathbf{p} near the Fermi surface can be written as

$$E_{\mathbf{p}} = \sqrt{v_F^2(|\mathbf{p}| - p_F)^2 + \Delta^2}, \quad (1)$$

where v_F and p_F are the Fermi-velocity and Fermi-momentum, respectively; and Δ is the energy gap, given by the standard equation [9],

$$1 = -V_0 \sum_{\mathbf{p}} \frac{1 - 2f_{\mathbf{p}}}{2E_{\mathbf{p}}}, \quad (2)$$

where V_0 is the (constant) pairing potential and

$$f_{\mathbf{p}} = \frac{1}{e^{E_{\mathbf{p}}/T} + 1} \quad (3)$$

is the Fermi-Dirac distribution function for Bogoliubov excitations.

If, however, the superfluid current is present ($\mathbf{V}_s \neq 0$) then fermions pair with momenta $(-\mathbf{p} + \mathbf{Q}, \mathbf{p} + \mathbf{Q})$ rather than with $(-\mathbf{p}, \mathbf{p})$, and the total momentum of a Cooper pair is

$$2\mathbf{Q} = 2m\mathbf{V}_s. \quad (4)$$

What will be the equation for the gap? The answer can be found in the paper by Bardeen [11] and is well known in the physics of superconductors. Now, instead of Eq. (2), one should write

$$1 = -V_0 \sum_{\mathbf{p}} \frac{1 - \mathcal{F}_{\mathbf{p}+\mathbf{Q}} - \mathcal{F}_{-\mathbf{p}+\mathbf{Q}}}{2E_{\mathbf{p}}}. \quad (5)$$

Here $\mathcal{F}_{\mathbf{p}+\mathbf{Q}}$ is the distribution function for Bogoliubov excitations with momentum $(\mathbf{p} + \mathbf{Q})$ in the system with non-zero \mathbf{V}_s ,

$$\mathcal{F}_{\mathbf{p}+\mathbf{Q}} = \frac{1}{e^{\mathfrak{E}_{\mathbf{p}+\mathbf{Q}}/T} + 1}, \quad (6)$$

where

$$\mathfrak{E}_{\mathbf{p}+\mathbf{Q}} \approx \frac{pQ}{m} + E_{\mathbf{p}} \quad (7)$$

is the energy of a Bogoliubov excitation with momentum $(\mathbf{p} + \mathbf{Q})$. In Eq. (7) we assumed $Q \ll p_F$ which is true in all interesting cases (see, e.g., [12–14]).

Eq. (5) can be written in terms of the quantity Δ_0 , which is the energy gap at $T = 0$ and $Q = 0$. It satisfies Eq. (2) with $f_{\mathbf{p}} = 0$. Using it, one can present Eq. (5) in the form

$$\frac{p_F m}{\pi^2} \ln \left(\frac{\Delta_0}{\Delta} \right) = \sum_{\mathbf{p}} \frac{\mathcal{F}_{\mathbf{p}+\mathbf{Q}} + \mathcal{F}_{-\mathbf{p}+\mathbf{Q}}}{E_{\mathbf{p}}}. \quad (8)$$

The solution to this equation gives the gap Δ as a function of T and $Q = |\mathbf{Q}|$.

First consider two limiting cases in which $\Delta(T, Q)$ vanishes.

(i) if $Q = 0$ then $\Delta = 0$ at

$$T = T_c \approx 0.567\Delta_0 \quad (\text{the well known BCS result}); \quad (9)$$

(ii) if $T = 0$ then $\Delta = 0$ at

$$Q \equiv Q_{\text{cr}0} = \frac{e}{2} \frac{\Delta_0 m}{p_F}. \quad (10)$$

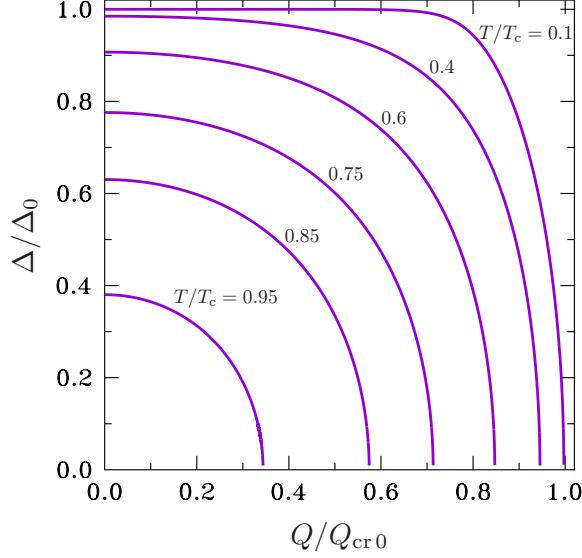


FIG. 1: (color online) The energy gap Δ (in units of Δ_0) versus $Q = mV_s$ [in units of Q_{cr0} , see Eq. (10)] for a set of temperatures $T/T_c = 0.1, 0.4, 0.6, 0.75, 0.85$, and 0.95 .

The latter result is less known but can be found, e.g., in [15]. Notice, that the well-known Landau criterion for superfluidity breaking gives $Q_{cr0}^{(\text{Landau})} = \Delta_0 m/p_F$ and is not accurate for a superfluid Fermi-liquid.

Some numerical solutions to Eq. (8) are presented in Figs. 1 and 2. Fig. 1 shows the gap $\Delta(T, Q)$ [in units of Δ_0] versus momentum Q [in units of Q_{cr0}] for a set of temperatures $T/T_c = 0.1, 0.4, 0.6, 0.75, 0.85$, and 0.95 . One sees that Δ is quite sensitive to variation of $Q = mV_s$ as long as $T \gtrsim 0.1T_c$. Another important conclusion that can be drawn from Fig. 1 is that (for a given T) the maximum critical momentum Q_{cr} strongly depends on temperature.

Fig. 2 illustrates this point more clearly. In the left panel we plot Q_{cr} (in units of Q_{cr0}) versus T (in units of T_c). The right panel shows the same dependence $Q_{cr}(T)$ but with Q_{cr} measured in units of

$$Q_{cr}^{(\text{app})}(T) \equiv \frac{e}{2} \frac{\Delta(T, 0) m}{p_F}. \quad (11)$$

We see that Q_{cr} changes with T in such a way that $Q_{cr}(T)/Q_{cr}^{(\text{app})}(T)$ is roughly constant.

Therefore, the energy gap Δ can be a strong function of the momentum $\mathbf{Q} = m\mathbf{V}_s$ or, in an arbitrary frame, a strong function of the difference $m(\mathbf{V}_s - \mathbf{V}_q) \equiv m\Delta\mathbf{V}$. We will refer to this effect as to the ‘ $\Delta\mathbf{V}$ -effect’. The critical value $\Delta V_{cr}(T)$ of $\Delta V = |\mathbf{V}_s - \mathbf{V}_q|$, at which

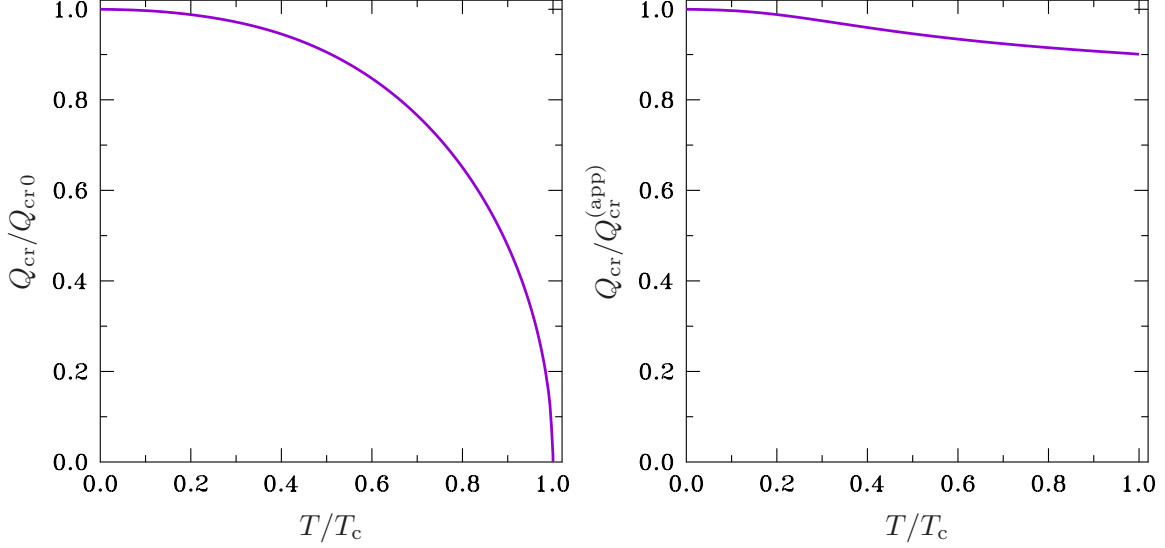


FIG. 2: (color online) Left panel: Q_{cr} (in units of $Q_{\text{cr}0}$) versus T (in units of T_c). Right panel: The same as in the left panel but Q_{cr} is in units of $Q_{\text{cr}}^{(\text{app})}$.

superfluidity dies out, is easily estimated by taking $Q_{\text{cr}} \sim Q_{\text{cr}}^{(\text{app})}$. Then, from Eq. (11), we obtain

$$\Delta V_{\text{cr}}(T) \sim 10^7 \left[\frac{\Delta(T, 0)}{10^9 \text{ K}} \right] \left(\frac{n_0}{n} \right)^{1/3} \text{ cm s}^{-1}, \quad (12)$$

where $\Delta(T, 0)$ is measured in Kelvins; $n_0 = 0.16 \text{ fm}^{-3}$ is the nucleon density in atomic nuclei; $n = p_F^3/(3\pi^2)$ is the particle number density.

III. IMPORTANCE OF THE ΔV -EFFECT FOR NEUTRON STARS

If the difference ΔV between the baryon superfluid velocities and a normal velocity is comparable to ΔV_{cr} , then the baryon energy gaps can be substantially reduced. A few interesting consequences of this ‘dynamical reduction’ of the gaps are discussed in the next section. Here we illustrate possible importance of the ΔV -effect by considering radial oscillations of a nonrotating superfluid NS whose core is composed of neutrons, protons, and electrons. The main question is at what oscillation amplitude ΔV becomes comparable to ΔV_{cr} ?

For simplicity we (i) assume that neutrons pair in the spin-singlet (1S_0) state [rather than in the triplet (3P_2) state] and (ii) neglect the Landau quasiparticle interaction between quasinucleons when calculating $\Delta_n(T, \mathbf{V}_{sn} - \mathbf{V}_q)$ [here and below the subscripts n and p refer

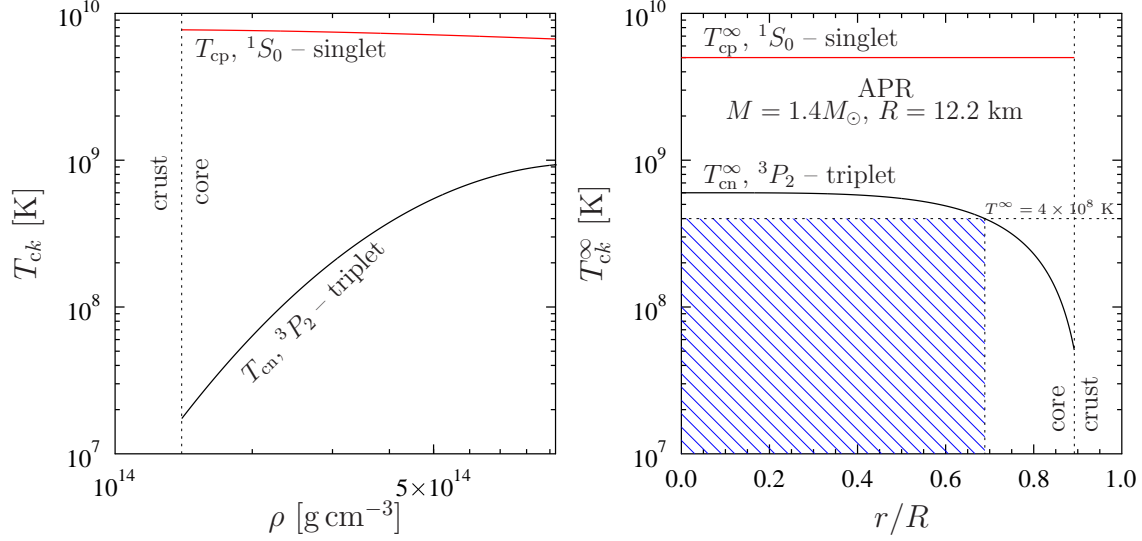


FIG. 3: (color online) Left panel: Nucleon critical temperatures T_{cn} and T_{cp} versus density ρ . Right panel: Redshifted critical temperatures T_{cn}^∞ and T_{cp}^∞ versus radial coordinate r (see text for details).

to neutrons and protons, respectively]. In [16] we outline the calculation of Δ_n and Δ_p as functions of $(\mathbf{V}_{sn} - \mathbf{V}_q)$ and $(\mathbf{V}_{sp} - \mathbf{V}_q)$ with allowance for interactions between quasiparticles.

The NS model used here and all the microphysics input are essentially the same as in [17]; we refer the reader to that work for more details. In particular, we consider the star of gravitational mass $M = 1.4M_\odot$, circumferential radius $R = 12.2$ km, central density $\rho_c = 9.26 \times 10^{14} \text{ g cm}^{-3}$, and adopt the APR EOS in the NS core [18]. The model of nucleon superfluidity employed here coincides with the model 3 of Ref. [17] and is shown in Fig. 3.

The left panel of Fig. 3 presents nucleon critical temperatures T_{cn} and T_{cp} versus density ρ in the NS core, the right panel demonstrates the red-shifted critical temperatures $T_{cn}^\infty \equiv T_{cn} e^{\nu/2}$ and $T_{cp}^\infty \equiv T_{cp} e^{\nu/2}$ (ν is the metric function) versus radial stellar coordinate r (in units of R). The redshifted proton critical temperature is taken to be constant $T_{cp}^\infty = 5 \times 10^9$ K; the redshifted neutron critical temperature varies with r and has maximum $T_{cn \text{ max}}^\infty = 6 \times 10^8$ K in the stellar centre. In the right panel of Fig. 3 we hatch the region occupied by the neutron superfluidity at a redshifted stellar temperature $T^\infty \equiv T e^{\nu/2} = 4 \times 10^8$ K.

To model oscillations of superfluid NSs one has to use the hydrodynamics of mixtures

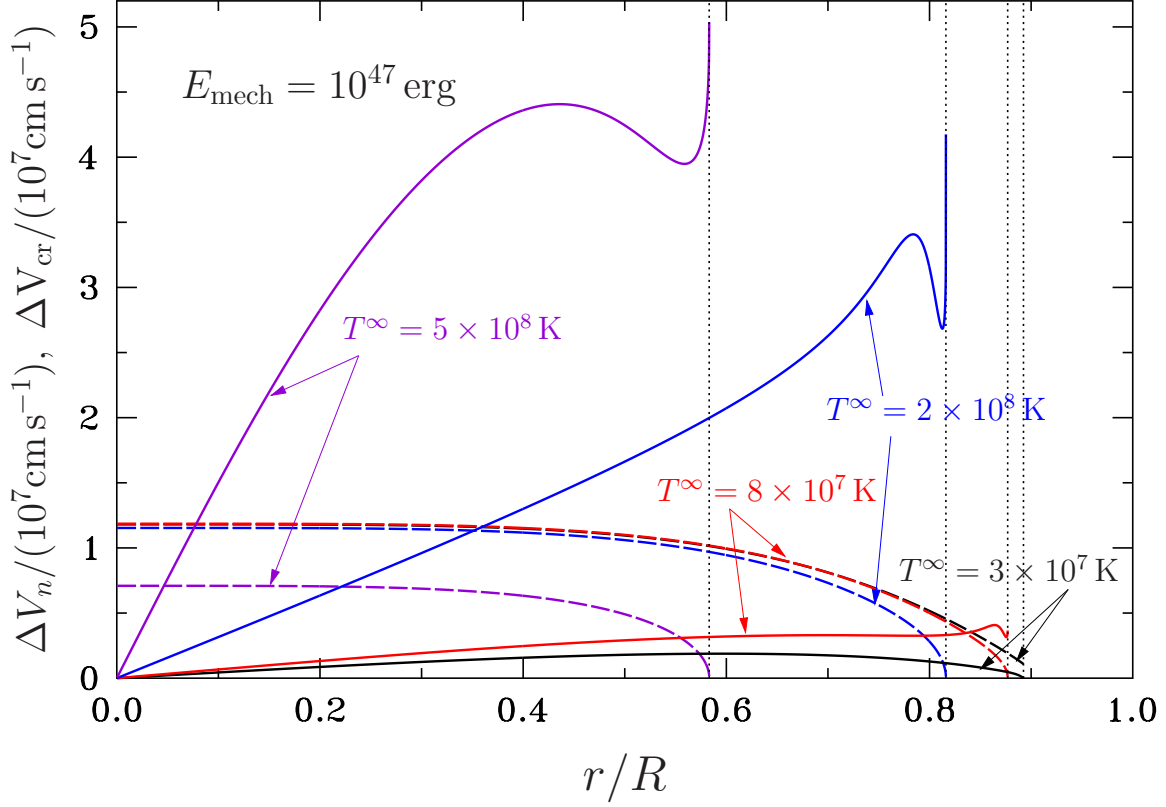


FIG. 4: (color online) Amplitudes of the eigenfunctions ΔV_n (solid lines) and the critical velocities ΔV_{cr} (dashes) versus r/R for the four temperatures $T^\infty = 3.0 \times 10^7$ K (black lines), 8.0×10^7 K (red lines), 2.0×10^8 K (blue lines), and 5.0×10^8 K (violet lines). To plot ΔV_n we assumed that the energy of oscillations is $E_{\text{mech}} = 10^{47}$ erg. The vertical dotted lines show the (temperature-dependent) boundaries between the inner superfluid and the outer normal regions. See text for details.

of superfluid Fermi-liquids [19–22]. The important parameter of such hydrodynamics is the so called entrainment matrix ρ_{ik} [12, 19] (or relativistic entrainment matrix Y_{ik} [13, 14]), which is very temperature-dependent [12, 14]. As a consequence, the eigenfrequencies and eigenfunctions of oscillating superfluid NS are also temperature-dependent [17, 21, 23]. Below we consider the *first* radial oscillation mode of a superfluid NS (see [17], particularly figure 3 there).

Figure 4 shows the amplitude of the eigenfunction $\Delta V_n \equiv |\mathbf{V}_{\text{sn}} - \mathbf{V}_{\text{q}}|$ and the critical velocity ΔV_{cr} as functions of r (solid and dashed lines, respectively; both in units of 10^7 cm s $^{-1}$). We plot ΔV_n and ΔV_{cr} for four redshifted stellar temperatures: $T^\infty = 3.0 \times 10^7$ K (black

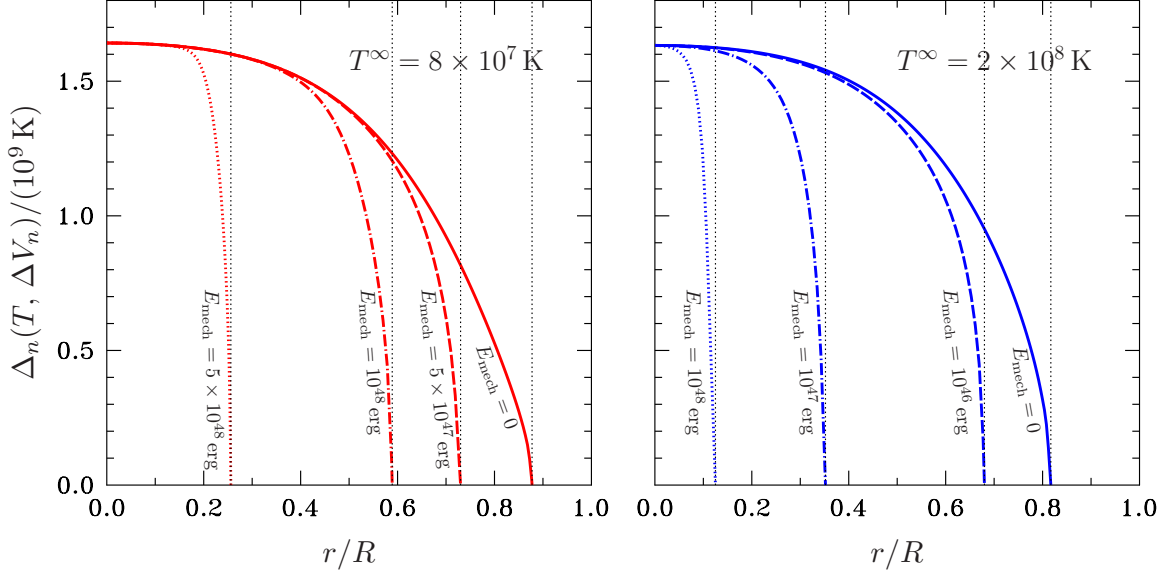


FIG. 5: (color online) Neutron energy gap $\Delta(T, \Delta V_n)$ (in units of 10^9 K) versus r/R for two temperatures $T^\infty = 8 \times 10^7$ K (left panel) and $T^\infty = 2 \times 10^8$ K (right panel) and some oscillation energies E_{mech} (indicated in the figure). Vertical dotted lines show r at which neutron superfluidity disappears ($\Delta_n = 0$). The larger E_{mech} the smaller the superfluid region. See text for details.

lines), 8.0×10^7 K (red lines), 2.0×10^8 K (blue lines), and 5.0×10^8 K (violet lines). The oscillation frequencies ω of the first radial mode for such temperatures are $\omega/(10^4 \text{ s}^{-1}) \approx 1.702, 1.702, 1.064$, and 0.516 , respectively.

The vertical dotted lines in Fig. 4 indicate (temperature-dependent) boundaries between the neutron superfluid region and the outer normal region with nonsuperfluid neutrons. In the normal region the functions ΔV_n and ΔV_{cr} are not defined. The oscillation energy of the star is $E_{\text{mech}} = 10^{47}$ erg. For a *nonsuperfluid* NS this energy corresponds to an oscillation amplitude

$$\varepsilon \equiv \lim_{r \rightarrow 0} \frac{\xi(r)}{r} \approx 4.4 \times 10^{-4}, \quad (13)$$

where $\xi(r)$ is the Lagrangian displacement [24].

It follows from Fig. 4 that ΔV_n can substantially exceed the critical values ΔV_{cr} , so that superfluidity is destroyed by oscillations in the large part of the stellar core (see the violet and blue curves). This means that the $\Delta \mathbf{V}$ -effect can greatly influence (or even drive) the dynamics of NSs already at rather modest oscillation amplitudes.

This point is additionally illustrated in Fig. 5, where we plot the neutron energy gap

TABLE I: Mechanical energy E_{mech} and the corresponding amplitude of oscillations ε , defined by Eq. (13).

$E_{\text{mech}}, \text{ erg}$	ε
0.0	0.0
10^{46}	1.4×10^{-4}
5.0×10^{46}	3.1×10^{-4}
10^{47}	4.4×10^{-4}
5.0×10^{47}	9.7×10^{-4}
10^{48}	1.4×10^{-3}
5.0×10^{48}	3.1×10^{-3}

$\Delta_n(T, \Delta V_n)$ versus r/R for two temperatures, $T^\infty = 8.0 \times 10^7$ K (left panel) and $T^\infty = 2.0 \times 10^8$ K (right panel), and a set of oscillation energies E_{mech} . In the left panel $\Delta_n(T, \Delta V_n)$ is shown for $E_{\text{mech}} = 0, 5.0 \times 10^{47}, 10^{48}$, and 5×10^{48} erg; in the right panel $\Delta_n(T, \Delta V_n)$ is shown for $E_{\text{mech}} = 0, 10^{46}, 10^{47}$, and 10^{48} erg. The oscillation amplitudes ε [given by Eq. (13)] for these oscillation energies are presented in Table.

Notice that in each panel of Fig. 5 the curves are plotted using the eigenfunctions $\Delta V_n(r)$, which differ from one another only by normalization (by the value of E_{mech}). For $E_{\text{mech}} = 10^{47}$ erg these eigenfunctions have already been presented in Fig. 4 (see the red and blue solid lines; the red line corresponds to $T^\infty = 8.0 \times 10^7$ K, the blue line – to $T^\infty = 2.0 \times 10^8$ K).

If $E_{\text{mech}} = 0$ (no oscillations; see the solid lines in both panels of Fig. 5) the gap Δ_n is unaffected by ΔV_n and is entirely determined by the dependence of T_{cn} on r (see Fig. 3). The vertical dotted lines in Fig. 5 indicate boundaries between the inner superfluid and the outer normal regions; these boundaries depend on E_{mech} . Obviously, the higher E_{mech} , the larger $\Delta V_n(r)$, and, correspondingly, the smaller the superfluid region and Δ_n . One sees that the gaps are very sensitive to variation of ΔV_n .

IV. POSSIBLE APPLICATIONS

As follows from the consideration of the previous section, the $\Delta\mathbf{V}$ -effect can operate at not too small oscillation amplitudes. All interesting consequences of this effect are related to the reduction of baryon gaps. Let us list some of them:

(1) The reduction of the gaps influences the entrainment matrix ρ_{ik} [12], which depends on them. As a result, ρ_{ik} will become a non-linear function of the oscillation amplitude. This will (i) make the oscillation equations nonlinear and hence (ii) affect the eigenfrequencies and eigenfunctions of oscillating NS. Moreover, this will (iii) influence the dissipation processes, because bulk viscosity terms explicitly depend on ρ_{ik} . In a rotating star the change (decrease) of the element ρ_{np} of the entrainment matrix will, in addition, (iv) decrease the mutual friction force [25], which is proportional to ρ_{np} . (We emphasize that the dependence of ρ_{np} on T and on ΔV_n and ΔV_p is a very important effect for mutual friction and related phenomena, which has been neglected in the literature.)

How to calculate the entrainment matrix ρ_{ik} taking into account the $\Delta\mathbf{V}$ -effect? A direct calculation is difficult (but one can make it in a manner similar to how it was done in [12]). A good approximation for ρ_{ik} could be to take the result of [12] and calculate ρ_{ik} with the velocity-dependent gaps from Sec. II instead of the gaps $\Delta_n(T, 0)$ and $\Delta_p(T, 0)$.

(2) Another important consequence of the $\Delta\mathbf{V}$ -effect is its impact on kinetic coefficients of NS matter, in particular, on the bulk and shear viscosities.

(i) *Bulk viscosity.* There are four bulk viscosity coefficients in the npe -matter of NSs [22]. All of them are generated by nonequilibrium beta-processes (direct or modified Urca reactions) and depend on the difference $\Delta\Gamma$ between the direct and inverse reaction rates. $\Delta\Gamma$ is generally a complicated function of T , Δ_n , Δ_p , and of the imbalance of chemical potentials $\delta\mu \equiv \mu_n - \mu_p - \mu_e$ [26, 27], where μ_i is the chemical potential for particle species $i = n, p, e$. Recently it has been shown [28], that if $\delta\mu > \max\{\Delta_n, \Delta_p\}$ then, even if $T \ll \Delta_n$ and/or Δ_p , the bulk viscosity is *not* suppressed by the nucleon superfluidity and can be very efficient. It seems that the $\Delta\mathbf{V}$ -effect of the reduction of the energy gaps Δ_n and Δ_p by relative motion of superfluid and normal component is *complementary* to the effect considered in [28]. Both effects act in unison to increase the bulk viscosity coefficients, and they are of comparable strength. Notice, however, that the effect of Ref. [28] can only affect the bulk viscosity coefficients, while the applicability range of the $\Delta\mathbf{V}$ -effect is wider;

it directly influences the baryon energy gaps and thus all dynamics of NSs.

(ii) *Shear viscosity.* Neglecting entrainment between baryon species ($\rho_{np} = 0$), the shear viscosity η can be calculated in the same fashion as was done, e.g., in [29] (the results will be the same). The only difference is that one should use the velocity-dependent gaps $\Delta_i(T, \Delta V_i)$ instead of $\Delta_i(T, 0)$ [$i = n, p$]. It is interesting that the $\Delta \mathbf{V}$ -effect can both increase or decrease the shear viscosity. For example, the electron shear viscosity η_e decreases with increasing ΔV_p (that is, with reducing Δ_p , because the electrons are better screened by protons when Δ_p is large [29]). On the other hand, the neutron shear viscosity η_n can either decrease or increase with growing ΔV_n and ΔV_p . The behaviour of η_n in that case is determined by the competition of two effects: by the increase of the normal density of neutron Bogoliubov excitations ρ_{qn} and by the reduction of the neutron mean free path λ due to more frequent collisions with neutron and proton Bogoliubov excitations (note that η_n can be estimated as $\eta_n \sim \rho_{qn} v_{Fn} \lambda$, where v_{Fn} is the neutron Fermi momentum). Similar effects were carefully analyzed in [30] in application to the neutron thermal conductivity.

An entrainment between neutrons and protons will strongly modify the derivation of the neutron shear viscosity, even neglecting the $\Delta \mathbf{V}$ -effect. The main difference will be the equilibrium Fermi-Dirac distribution function for neutron Bogoliubov excitations in a system with superfluid currents. This function was first obtained in [12] [see equation (28) there]; it is very different from the standard expression, valid when $\rho_{np} = 0$. To our best knowledge, a derivation of η_n in a system *with* entrainment has not been attempted in the literature.

(3) Finally, there is a number of important consequences of the fact that the relative velocity $\Delta \mathbf{V}$ between the superfluid and normal liquid components cannot be too large in a *stationary* rotating NS. Here we present two of them.

(i) It is generally accepted that neutron vortices are pinned to atomic nuclei in the NS crust (or to magnetic flux tubes in the NS core). At some critical $\Delta \mathbf{V}$ they can unpin from the nuclei (or from magnetic flux tubes). However, in some models (e.g., [31]) pinning is so strong that the critical relative velocity can be as high as $10^6 \div 10^7$ cm s⁻¹. These values are close to ΔV_{cr} [see Eq. (12)]. Thus, the $\Delta \mathbf{V}$ -effect can be very important for such models.

(ii) In Refs. [32, 33] a two-stream instability is discussed that can be triggered once the relative velocity $\Delta \mathbf{V}$ reaches some critical value. According to [32, 33], this critical value is of the order of the sound speeds, i.e., it is *much greater* than the typical ΔV_{cr} , at which

superfluidity completely disappears [see Eq. (12)]. In other words, it is not very probable that this instability is realized in NSs.

V. CONCLUSION

The baryon energy gaps depend on the relative velocity between the superfluid and normal components ($\Delta\mathbf{V}$ -effect). We propose, for the first time, that this effect may have a strong impact on the dynamical properties of NSs. We illustrate this point by considering radial oscillations of an NS with superfluid nucleon core and a nonsuperfluid crust. However, we stress that the $\Delta\mathbf{V}$ -effect should be equally important in the crust of NSs where superfluid neutrons are present, as well as in the interiors of hyperon and quark stars. Although we discussed some immediate applications in Sec. IV, it is clear that more efforts are needed to analyze all possible consequences of this effect on the evolution of NSs.

Acknowledgments

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